Section 5.6: The Area Between Two Curves (Minimum Homework: 1 – 19 odds)

In section 5.6 we will learn how to calculate the area between the graphs of two functions.

Let me try to explain the theory in this section with a few graphs.

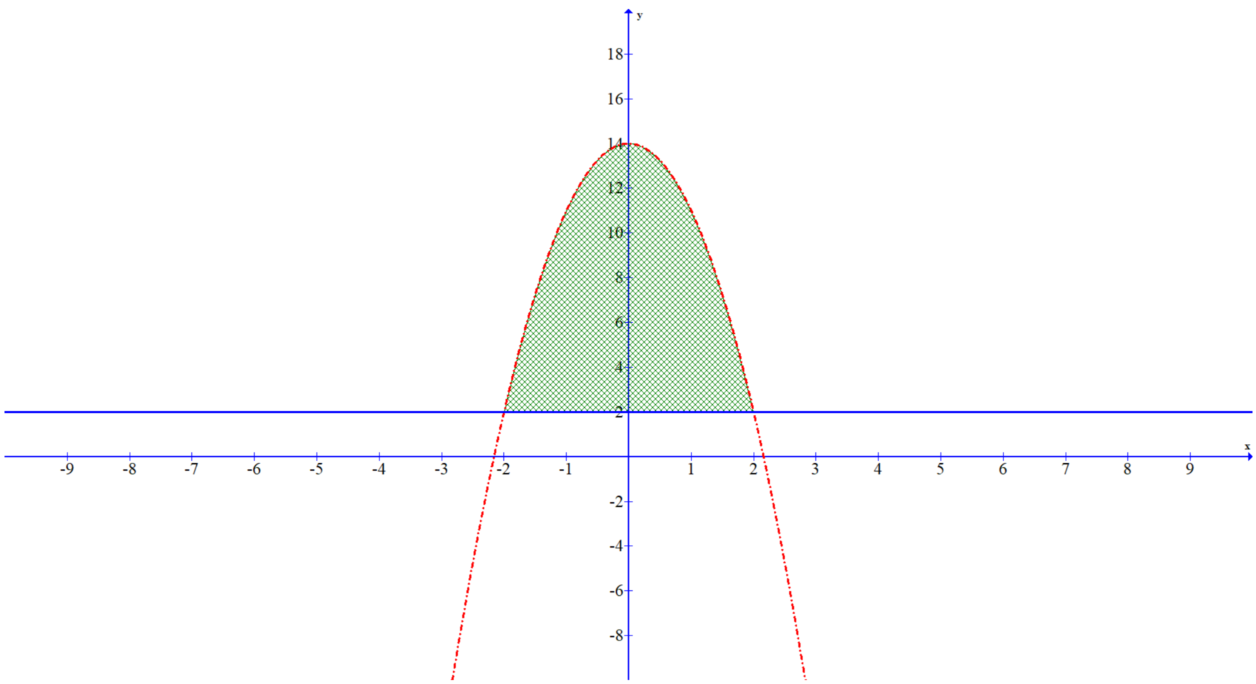
Calculator the shaded are between the functions.

$f(x) = 14 - 3x^2$

and the function

$g(x) = 2$

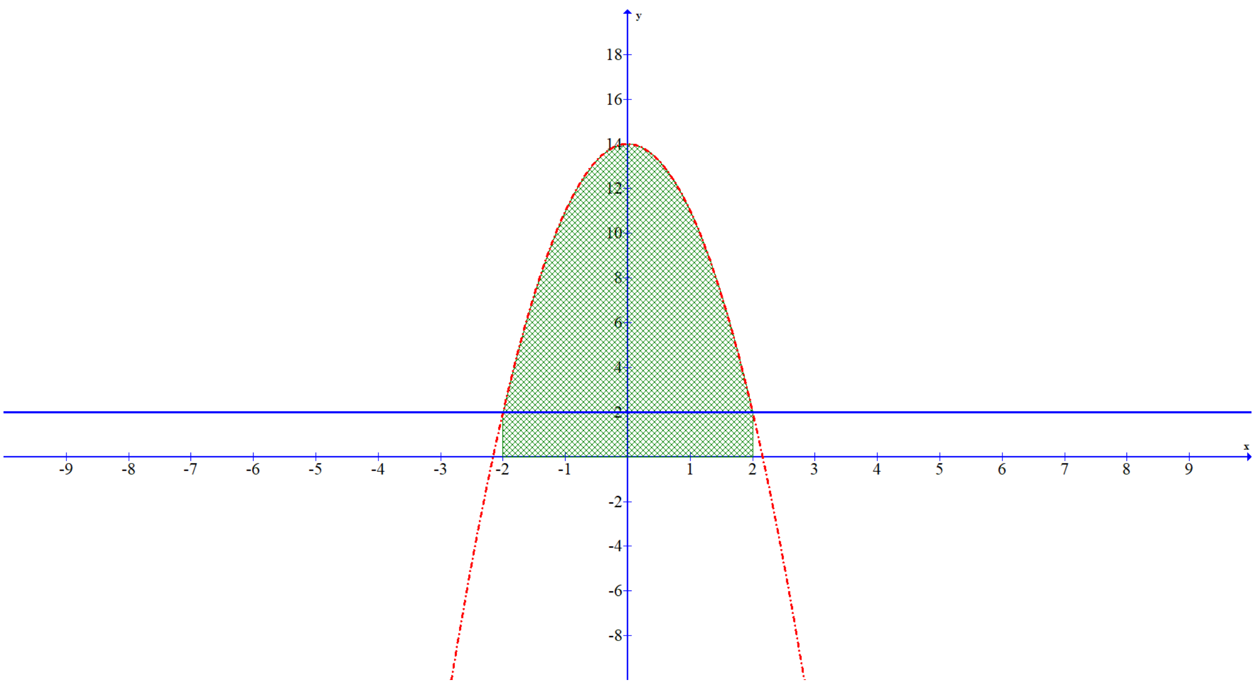
over the interval $[-2,2]$



We can find the shaded area by computing two areas and subtracting the results.

$\int_{-2}^{2}(14 - 3x^2)dx$

This will give the area down to the x-axis. It will over-estimate the area. As it includes a bit extra. The extra area is beneath the graph of $g(x) = 2$ and the x-axis.

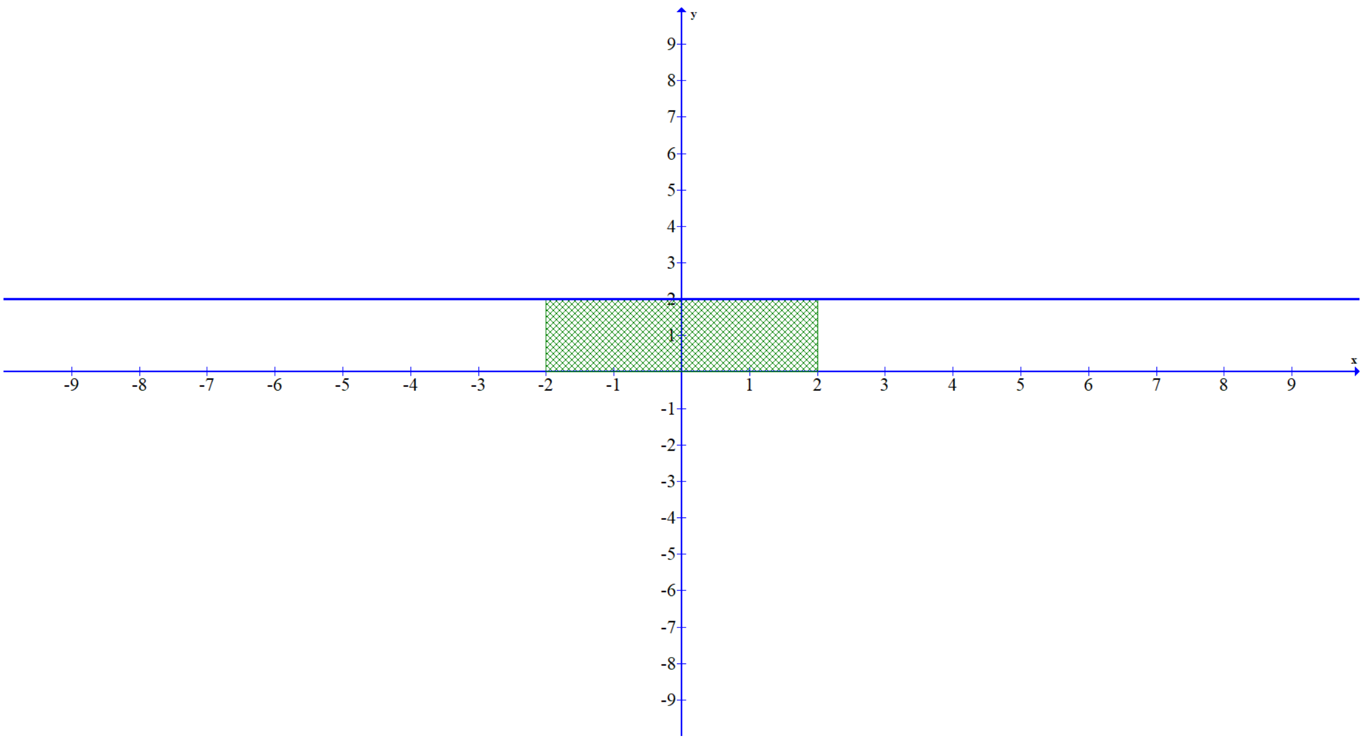


This area is 40 square units

We need to determine the excess area and subtract it out.

This is the integral that will calculate the excess area.

$\int_{-2}^{2} 2dx$



This area is $8\ square\ units$

The shaded area in the original graph = $40 - 8 = 32$

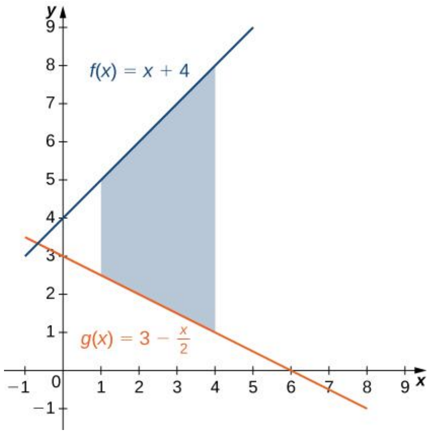
Area between curves over an interval $[a, b] = \int_a^b (top\ function - bottom\ function)dx$

If we allow the top function graphed to be called $f(x)$ And the bottom function graphed to be called $g(x)$ Then the area between the function $f(x) and\ g(x)\ over\ the\ interval\ [a, b]$ can be found by evaluating this definite integral: $\int_a^b \left(f(x) - g(x)\right)dx$

I could find the shaded area in the first graph directly by applying the theorem above.

Integral to calculate original shaded area = $\int_{-2}^{2} (14 - 3x^2 - 2)dx = 32$

For Example:
a) Create the integral needed to find the shaded area
b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)



We need to integrate between x = 1 and x = 4 as this is the region that is shaded.
We need to subtract $f(x) - g(x)$ *in the integral as* $f(x)$ *is the top function*

Required integral:
$$a)\ \int_1^4 \left((x+4) - \left(3 - \frac{x}{2}\right)\right) dx \ = \int_1^4 \left(1x + 4 - 3 + \frac{1}{2}x\right) dx \ = \int_1^4 \left(\frac{3}{2}x + 1\right) dx$$

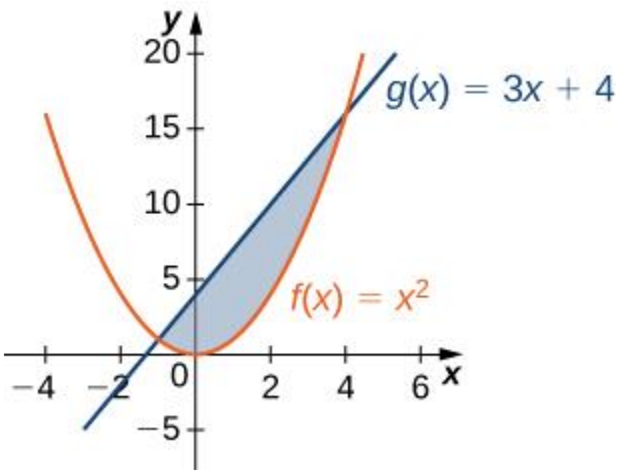
Each of these 3 integrals will give the same answer. I do not really need to simplify if I am using my calculator to find the area.

b) shaded area = 14.25

For Example:

a) Create the integral needed to find the shaded area

b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)



We need to integrate between x = -1 and x = 4 as this is the region that is shaded.

We need to subtract g$(x) -$
$f(x)$ *in the integral as* $g(x)$*is the top function*

a) Required integral:

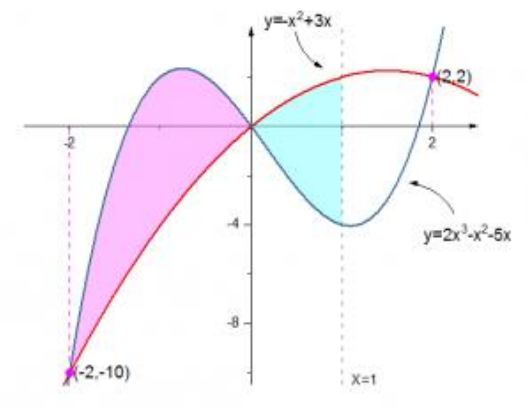
$\int_{-1}^{4}(3x + 4 - x^2)dx = \int_{-1}^{4}(-1x^2 + 3x - 4)dx$

I did not use any extra parenthesis in the first integral as $f(x)$ only has a single term.

Both integrals should give the same answer.

b) shaded area 20.83 (rounded to 2-decimal places)

For Example:
For Example:
a) Create the integral needed to find the shaded area
b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)



a) In this example the graphs cross over the shaded region. We must calculate each are separately and then add the two areas together to find the shaded area.

In the purple region the $y = 2x^3 - x^2 - 5x$ is the top function. It goes first in the integral.

Purple area: $\int_{-2}^{0}\left((2x^3 - x^2 - 5x) - (x^2 + 3x)\right)dx =$
$\int_{-2}^{0}(2x^3 - 1x^2 - 5x - 1x^2 - 3x)dx$

$= \int_{-2}^{0}(2x^3 - 2x^2 - 8x)dx$

All three of these integrals should give the same result.

Purple area = 2.67

In the blue region the $y = x^2 + 3x$ is the top function. It goes first in the integral.

Blue area: $\int_0^1 \left((x^2 + 3x) - (2x^3 - x^2 - 5x)\right)dx$
$=$
$\int_0^1 (1x^2 + 3x - 2x^3 + 1x^2 + 5x)dx$

$= \int_0^1 (-2x^3 + 2x^2 + 8x)dx$

All three of these integrals should give the same result.

a) $\int_{-2}^0 (2x^3 - 2x^2 - 8x)dx$ + $\int_0^1 (-2x^3 + 2x^2 + 8x)dx$

Blue area = 4.17

b) Total combined shaded area = 2.67 + 4.17

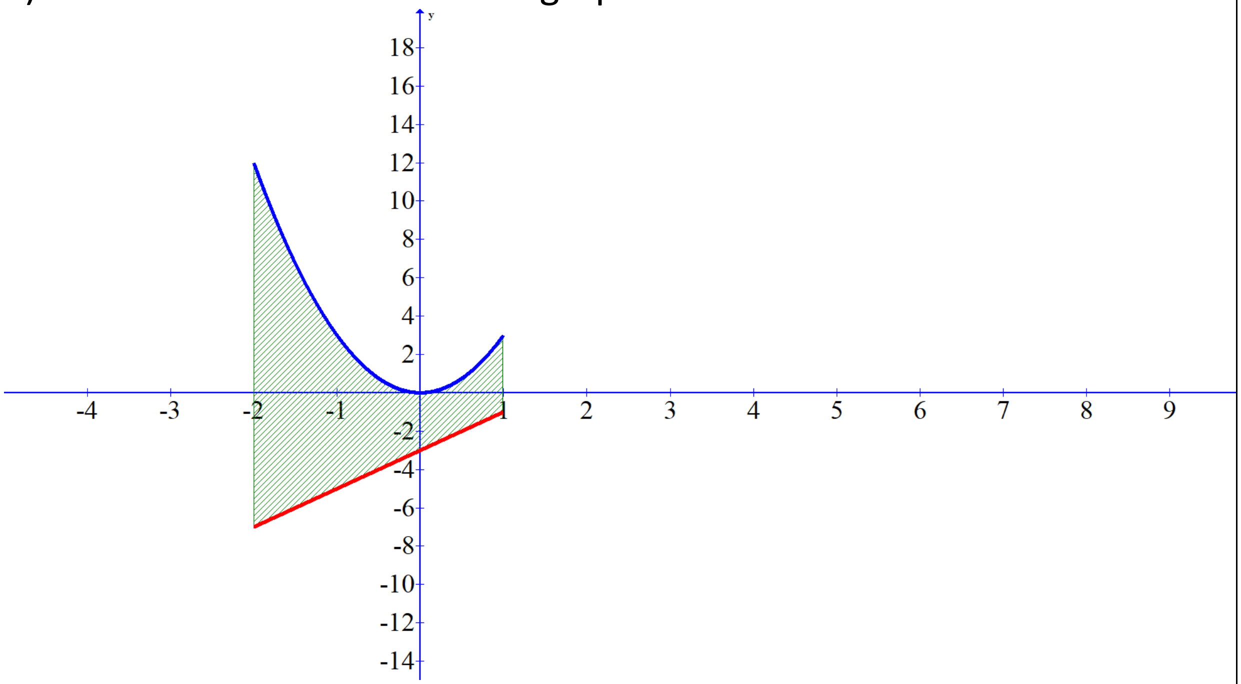
Answer: Total shaded area = 6.84

We need to be able to calculate an area between two graphs when we are not given a graph to work from.

Example: Use the functions to answer the following.

$f(x) = 2x - 3$ and $g(x) = 3x^2$ on $[-2,1]$.

a) Use a calculator to sketch a graph of both functions.



b) Determine the function that is the "top" function.

$g(x) = 3x^2 is\ the\ top\ funcion$

c) Create the integral needed to find the area between the curves.

$\int_{-2}^{1}\left(3x^2 - (2x - 3)\right)dx \ = \int_{-2}^{1}(3x^2 - 2x + 3)dx$

d) Find the area between the graphs over the given interval [-2,1] (You may use your calculator to compute the desired area.)

Desired area = 21

Homework:

#1 – 10:

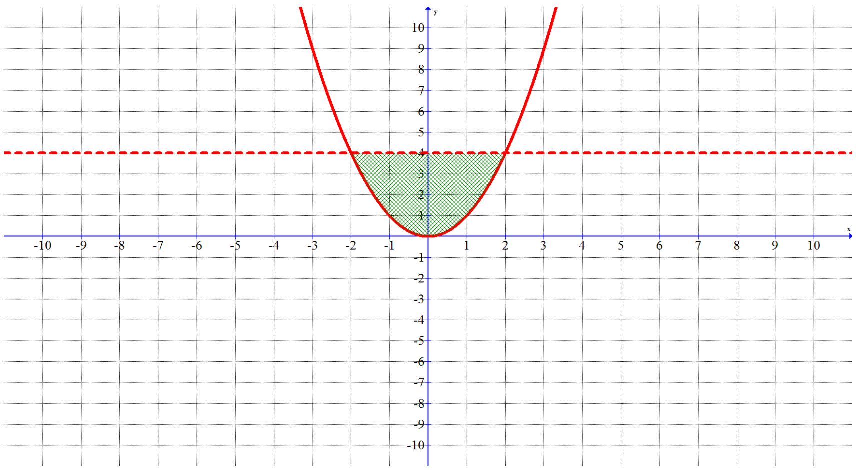
a) Create the integral needed to find the shaded area

b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)

1)

The function whose graph is represented by the dashed is f(x) = 4

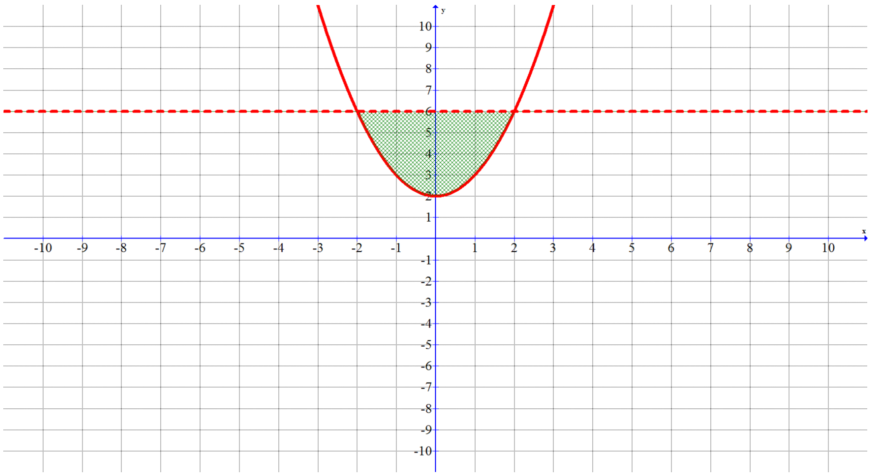
The function whose graph is represented by the solid line is g(x) = x^2



2)

The function whose graph is represented by the dashed is f(x) = 6

The function whose graph is represented by the solid line is g(x) = $x^2 + 2$



a) Create the integral needed to find the shaded area

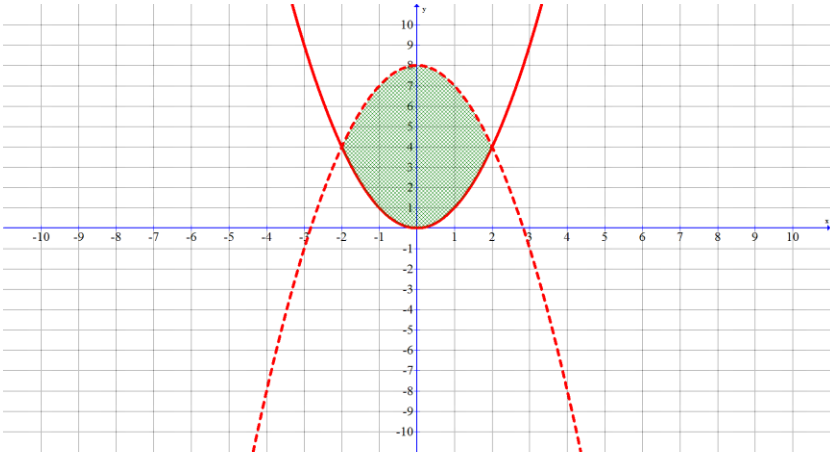
b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)

2a) $\int_{-2}^{2}(-x^2 + 4)dx$ 2b) Answer: 10.67

3)

The function whose graph is represented by the dashed is $f(x) = -x^2 + 8$

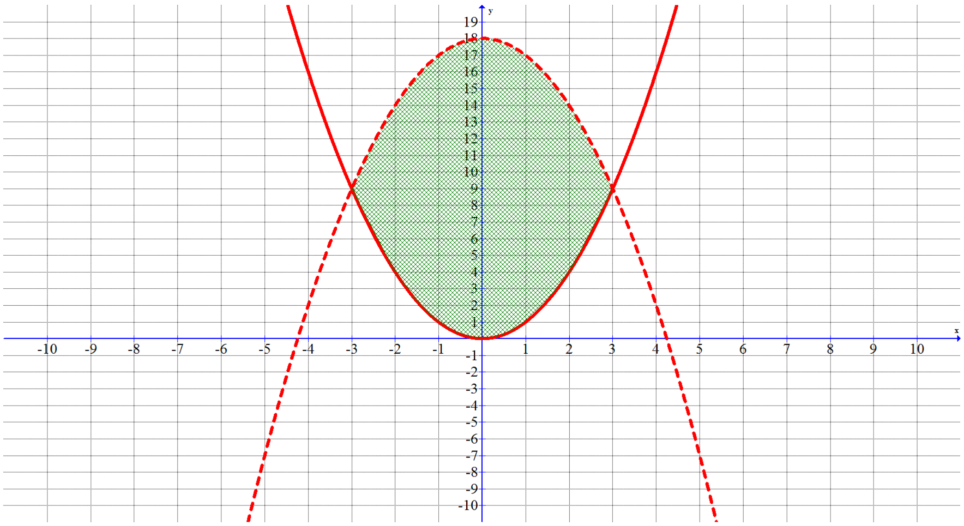
The function whose graph is represented by the solid line is $g(x) = x^2$



4)

The function whose graph is represented by the dashed is f(x) = $-x^2 + 18$

The function whose graph is represented by the solid line is g(x) = x^2



a) Create the integral needed to find the shaded area

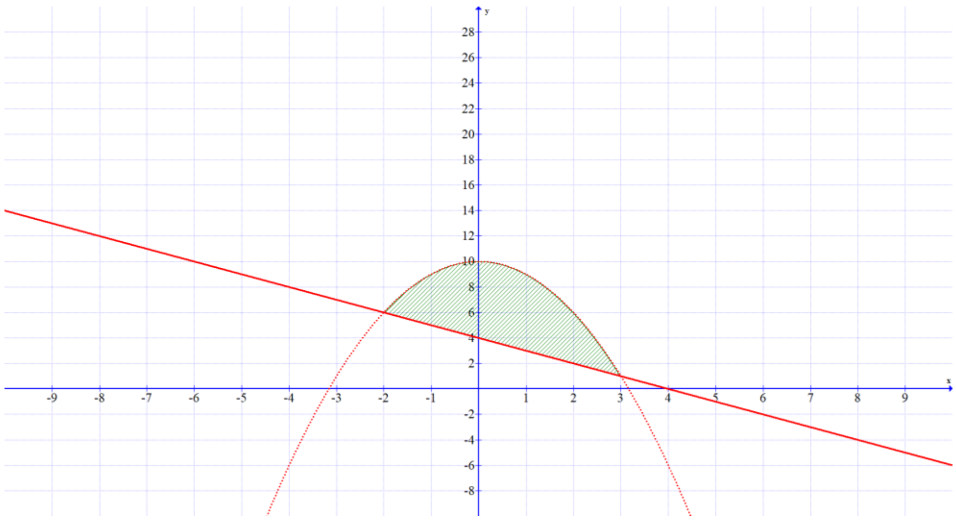
b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)

4a) $\int_{-3}^{3}(-2x^2 + 18)dx$ 4b) Answer: 72

5)

The function whose graph is represented by the dashed is $f(x) = -x^2 + 10$

The function whose graph is represented by the solid line is $g(x) = -x + 4$



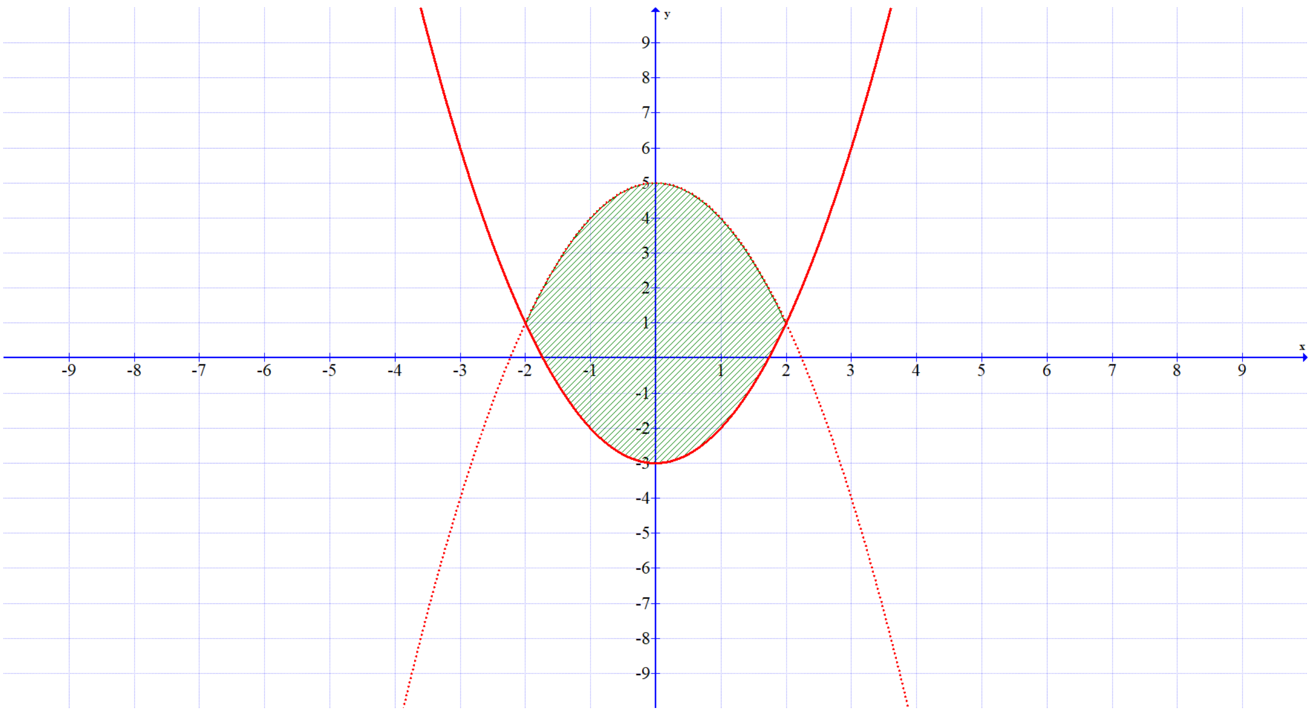
6)

The function whose graph is represented by the dashed is

$$f(x) = -x^2 + 5$$

The function whose graph is represented by the solid line is

g(x) = $x^2 - 3$



a) Create the integral needed to find the shaded area

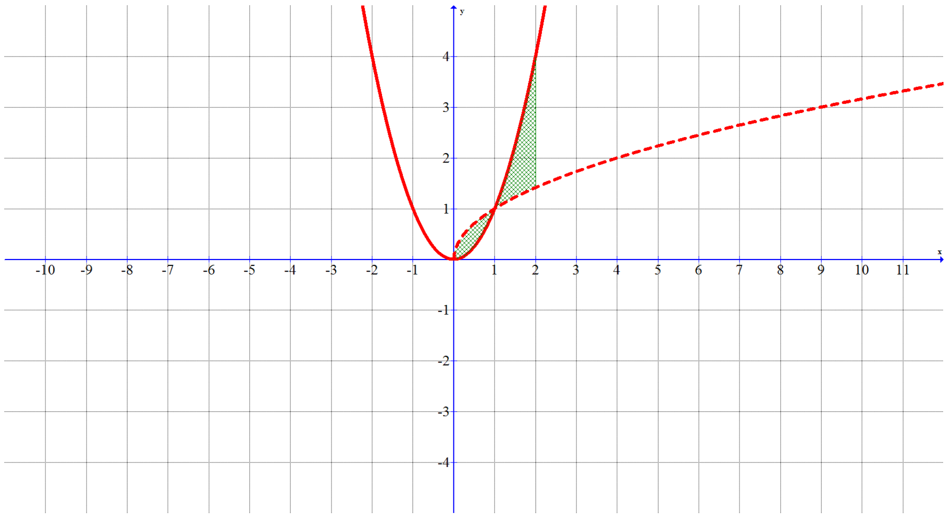
b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)

6a) $\int_{-2}^{2}(-2x^2 + 8)dx$ 6b) Answer: 21.33

7)

The function whose graph is represented by the dashed is $f(x) = \sqrt{x}$

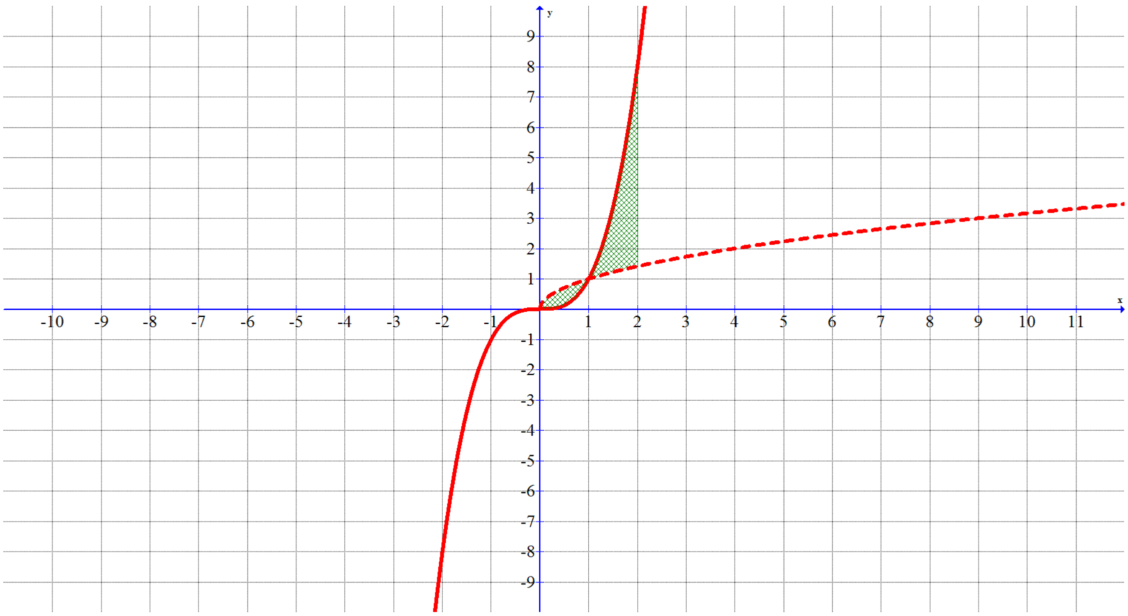
The function whose graph is represented by the solid line is $g(x) = x^2$



8)

The function whose graph is represented by the dashed is $f(x) = \sqrt{x}$

The function whose graph is represented by the solid line is $g(x) = x^3$



a) Create the integral needed to find the shaded area

b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)

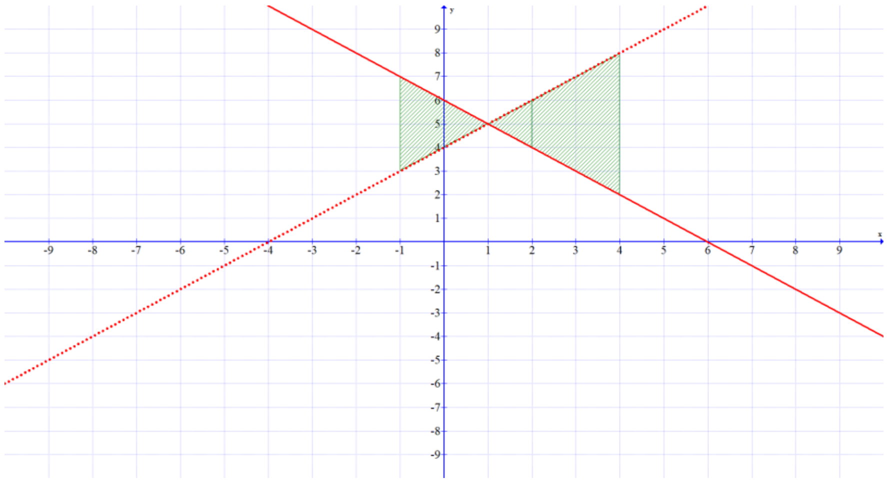
8a) $\int_0^1 (\sqrt{x} - x^3)dx + \int_1^2 (x^3 - \sqrt{x})dx$

8b) Answer: $area = 0.42 + 2.53 = 2.95$

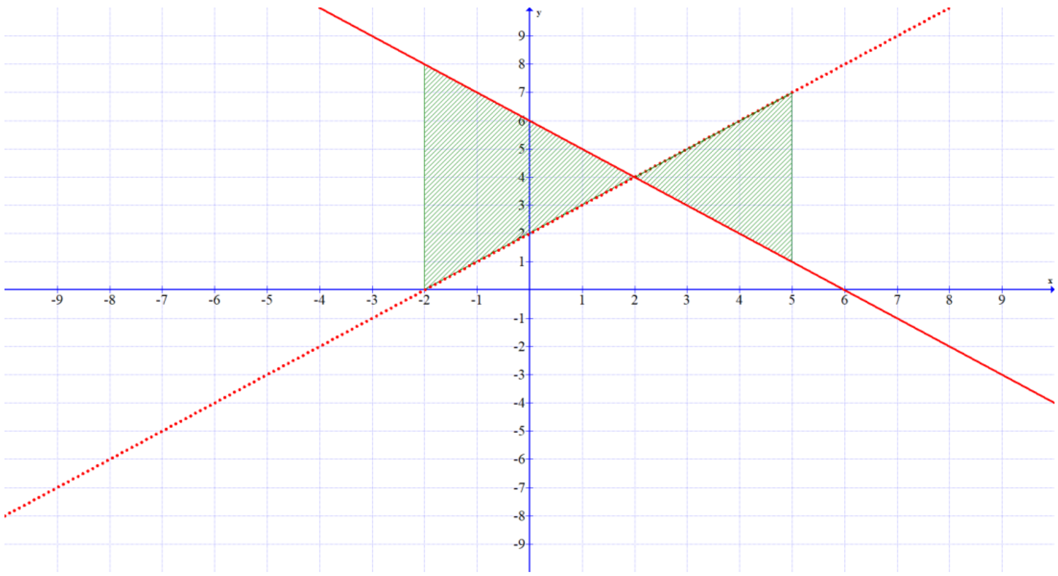
9)

The function whose graph is represented by the dashed is $f(x) = x - 6$

The function whose graph is represented by the solid line is $g(x) = 6 - x$



10)



The function whose graph is represented by the dashed is $f(x) = x + 2$

The function whose graph is represented by the solid line is $g(x) = 6 - x$

a) Create the integral needed to find the shaded area

b) Find the shaded area. Round to 2 decimals as needed. (you may use your calculator to determine the area)

10a) $\int_{-2}^{2}(-2x+4)dx + \int_{2}^{5}(2x-4)dx$

10b) Answer: $area = \ 16 + 9 = 25$

#11-16:

a) Use a calculator to sketch a graph of both functions. (You do not need to copy the graph on paper.)

b) Determine the function that is the "top" function.

c) Create the integral needed to find the area between the curves.

d) Find the area between the graphs over the given interval [a,b]

(You may use your calculator to compute the desired area.)

11) $f(x) = x + 1 \text{ and } g(x) = 7 - x \text{ on } [0,3]$.

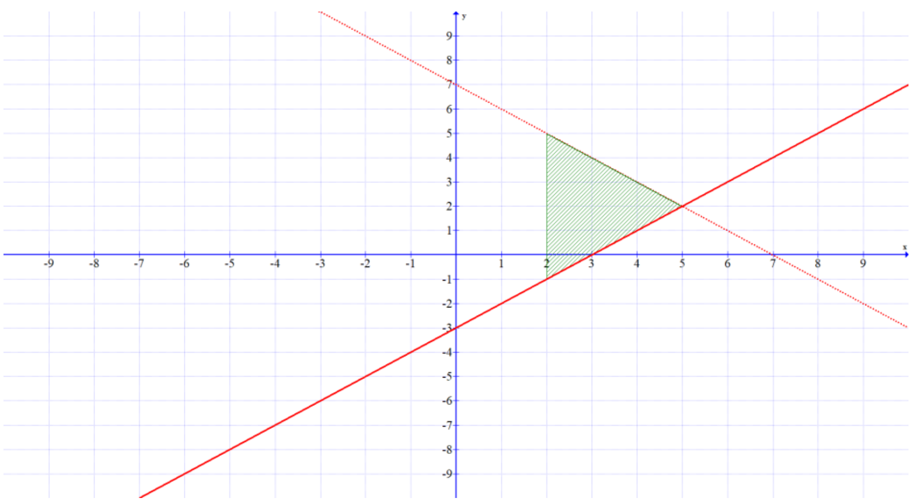
12) $f(x) = x - 3 \text{ and } g(x) = -x + 7 \text{ on } [2,5]$.

12a) Use a calculator to sketch a graph of both functions

(You do not need to copy the graph on paper.)

$f(x) = x - 3 \text{ is the dashed curve}$

$g(x) = -x + 7 \text{ is the solid curve}$



12b) Determine the function that is the "top" function.

12c) Create the integral needed to find the area between the curves.

12d) Find the area between the graphs over the given interval [a,b]

(You may use your calculator to compute the desired area.)

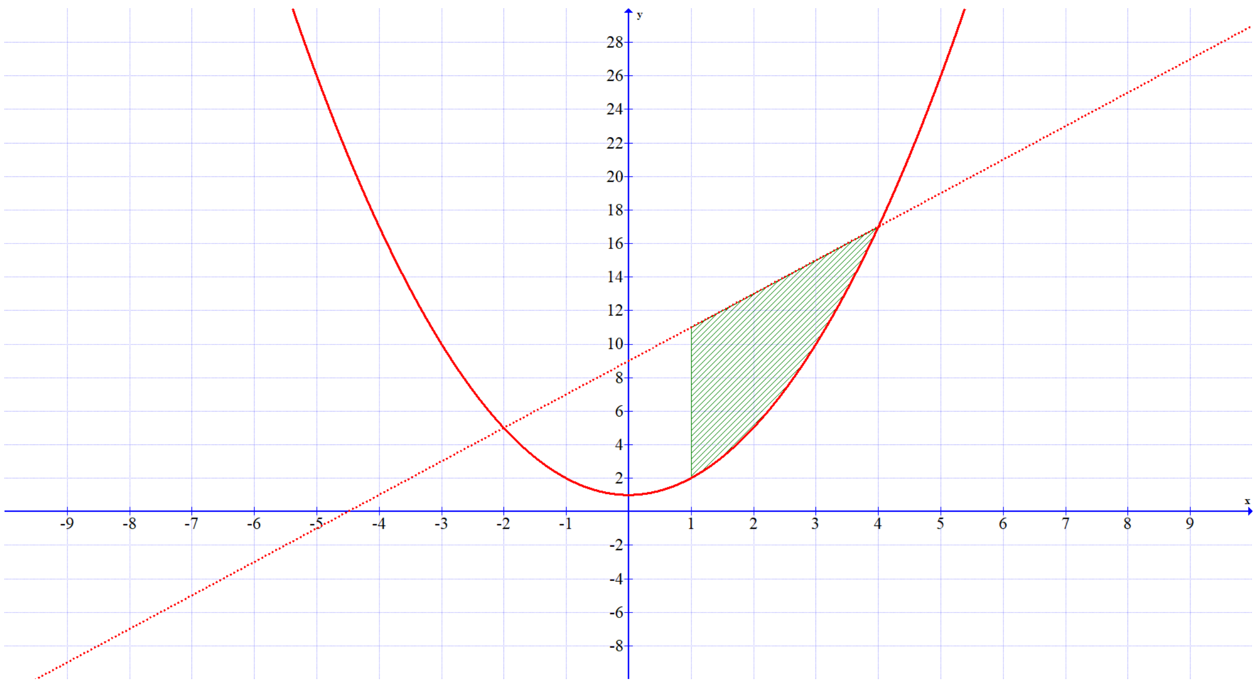
$(answer\ \ 9)$

13) $f(x) = 4x + 16 \text{ and } g(x) = 2x^2 + 10 \text{ on } [-1,3]$.

14) $f(x) = 2x + 9 \text{ and } g(x) = x^2 + 1 \text{ on } [1,4]$.

14a) Use a calculator to sketch a graph of both functions.
(You do not need to copy the graph on paper.)

$f(x) = 2x + 9 \;\; dashed \;\; g(x) = x^2 + 1 \;\; solid$



14b) Determine the function that is the "top" function.

14c) Create the integral needed to find the area between the curves.

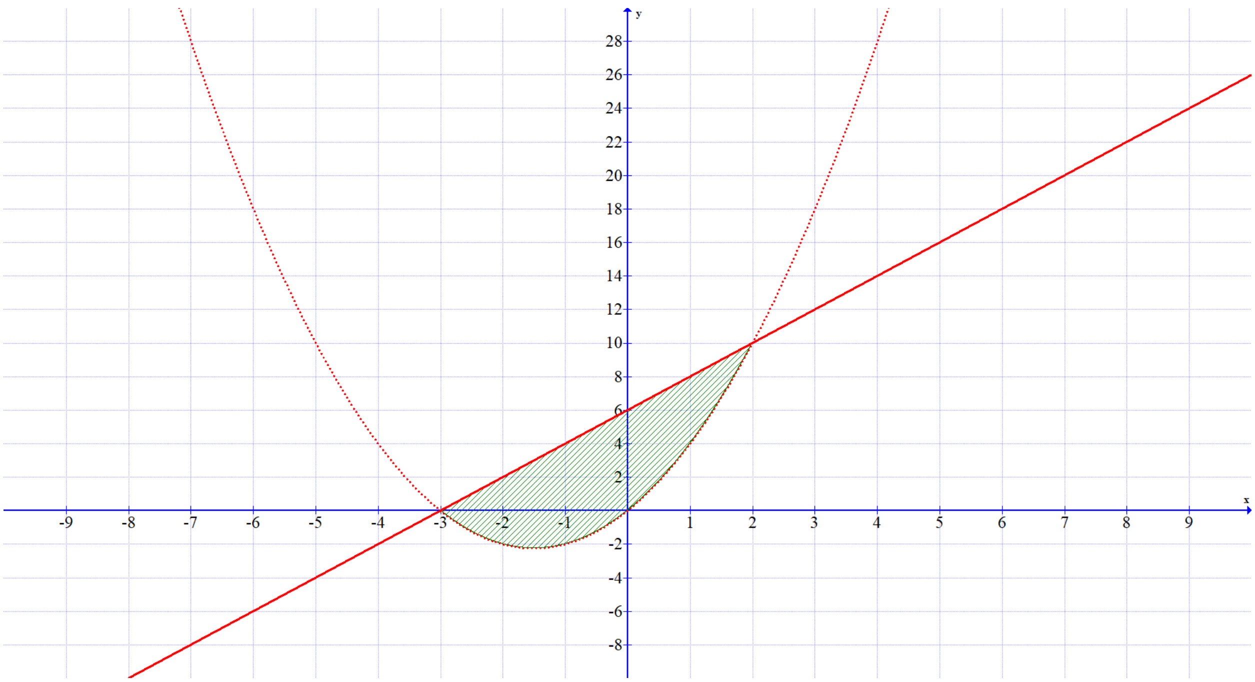
14d) Find the area between the graphs over the given interval [a,b]
(You may use your calculator to compute the desired area.)
$answer$: 18

15) $f(x) = x^2 + 6\ and\ g(x) = x + 8\ on\ [-1,2]$.

16) $f(x) = x^2 + 3x\ and\ g(x) = 2x + 6\ on\ [-3,2]$.

16a) Use a calculator to sketch a graph of both functions.
(You do not need to copy the graph on paper.)

$f(x) = x^2 + 3x\ dashed\ \ g(x) = 2x + 6\ solid$



16b) Determine the function that is the "top" function.

16c) Create the integral needed to find the area between the curves.

16d) Find the area between the graphs over the given interval [a,b]
(You may use your calculator to compute the desired area.)
$answer$: 20.83